

Maths problems Set 0.2 (series)

August 2015

1. Show that

$$\sum_{k=1}^n k = \frac{n(n+1)}{2} \quad \text{and hence that} \quad \sum_{l=0}^{n-1} 2(2l+1) = 2n^2$$

2. Use the binomial series expansion to find the coefficients in the expansion of $(1+x)^7$
3. The binomial series can be used to expand $f(x) = (1+x)^r$, where r is not necessarily an integer, as long as $-1 < x < 1$.
Find the the first 3 terms and the general term of the binomial expansions of $(1+x)^{-1}$, $\sqrt{1-x}$ and $(1-x)^{0.1}$. Then deduce a general expression to approximate $(1+x)^r$ when $|x| \ll 1$
4. Find the first 4 terms and the general term in the Taylor series expansion for e^x about $x = 0$
5. Find the first 4 terms and the general term in the Taylor expansion for $f(x) = (1-x)^{-1}$ about $x = 0$ and identify the range of x in which the series is valid
6. Find the first 4 terms of the Taylor expansion of $(1+x)^r$ near $x = 0$
7. Q: Find the first 4 terms and the general term in the Taylor expansions of $\sin x$ and $\cos x$ near $x = 0$
8. What is the general term of the Taylor expansion of $f(x + \varepsilon)$, where ε is a small displacement from $x = a$?
9. Find a third-order polynomial that approximates the function $f(x) = e^{2x} \sin(3x)$ at small values of x .
10. A particle moves in one dimension in a potential $V(x)$ which has a minimum at $x = x_0$. find an approximate equation of motion appropriate to small displacements from equilibrium